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THE CONVERGENCE OF AN INSTRUMENTAL-VARIABLE LIKE RECURSION.(U)

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THE CONVERGENCE OF AN INSTRUMENTAL-VARIABLE LIKE RECURSION

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V. Solo

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53706

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ABSTRACT

It is proved that a modified form of the IV recursion, for the parameters of a scalar Transfer Function Time Series model, converges provided a certain positive real condition holds. The result is disappointing because the positive real condition depends on the Transfer Function characteristic function.

AMS (MOS) Subject Classifications: 62M10, 93B30

Key Words: Stochastic approximation, Strong convergence, Kalman Filter, Recursive least squares.

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SIGNIFICANCE AND EXPLANATION

For the prediction of signals in real time it is often necessary to estimate the parameters of a signal model in real time also. A large number of these so called recursive parameter estimation schemes have been suggested and the basic question for these schemes is whether or not they converge and whether they have to be monitored to ensure they do. This work deals with the convergence analysis of one (of only two known) particular recursion(s) that does not require monitoring.

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The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

THE CONVERGENCE OF AN INSTRUMENTAL-VARIABLE LIKE RECURSION

V. Solo

I. Introduction. Recently the author (Solo, 1979) has proved the convergence of the AML recursion (provided a certain positive real condition holds) without requiring the recursion be monitored - i.e. projected into a stability region should it become unstable. In this article it is shown there is another recursion that converges without being monitored (but provided a certain positive real condition holds) namely a modified form of the IV (Instrumental Variable) recursion of Young (1974, 1976).

Consider the following TFWN (Transfer function white noise) model which is defined by a formula for its innovations sequence as

$$e_n(\underline{\theta}) = y_n - s_n(\theta) = y_n - (1+A(L))^{-1}B(L)u_n$$

where y_n, $s_n(\cdot)$, u_n are observed, output, input sequences respectively while

$$B(L) = \sum_1^{n_b} b_i L^i$$

etc. and L is the lag or inverse z transform operator. Also $\underline{\theta} = (a_1,\ldots,a_{n_a}\ b_1,\ldots,b_{n_b})'$ and $\underline{\theta} \in R$ a compact subset of the open set

$$S = \{\underline{\theta} \mid 1+A(L) \text{ has all zeroes outside the unit circle in the complex } L \text{ plane}\} \quad (1)$$

It is assumed there is a true value $\underline{\theta}_0 \in R$ so that $e_n(\underline{\theta}_0)$ (denoted ϵ_n) is a nonlinear innovations sequence (i.e., ϵ_n is orthogonal to, or uncorrelated with, any function of the past (of ϵ_n) linear or nonlinear). Thus $E(\epsilon_n \mid F_{n-1}) = 0$ where F_{n-1} is the "history" or increasing σ-algebras generated by ϵ_n: ϵ_n is also called a martingale difference sequence. It

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is also assumed ϵ_n has a constant prediction variance i.e., $E(\epsilon_n^2 | F_{n-1}) = \sigma^2$. The ergodicity assumption then ensures

$$n^{-1}\sum_1^n \epsilon_k^2 \to \sigma^2 \qquad \text{a.s.} \tag{2}$$

The u_k sequence may be deterministic or stationary ergodic (independent of the ϵ_n sequence) and is assumed to have a finite non zero power

$$0 < \lim n^{-1}\sum_1^n u_k^2 = \sigma_u^2 < \infty \quad . \tag{3}$$

The IV recursion for $\underline{\theta}$ is

$$\hat{\underline{\theta}}_n = \hat{\underline{\theta}}_{n-1} + \underline{P}_n \, \breve{\underline{\varphi}}_n \, e_n$$

$$\underline{P}_n^{-1} = \underline{P}_{n-1}^{-1} + \breve{\underline{\varphi}}_n \, \underline{z}_n'$$

or $$\underline{P}_n = \underline{P}_{n-1} - \underline{P}_{n-1} \, \breve{\underline{\varphi}}_n \, \underline{z}_n' \, \underline{P}_{n-1}/(1 + \breve{\underline{\varphi}}_n' \, \underline{P}_{n-1} \, \underline{z}_n)$$

$$e_n = y_n = \breve{s}_n$$

where $$\breve{\underline{\varphi}}_n' = (-\breve{s}_{n-1} \ldots -\breve{s}_{n-n_a} \; u_{n-1} \ldots u_{n-n_b})$$

$$\breve{s}_n = -\breve{\underline{\varphi}}_n' \, \underline{\theta}_{n-1}$$

$$\underline{z}_n' = (-y_{n-1} \ldots -y_{n-n_a} \; u_{n-1} \ldots u_{n-n_b}) \quad .$$

The so-called symmetric IV recursion replaces $\underline{z}_n$ above by $\breve{\underline{\varphi}}_n$. The resulting recursion can also be obtained by applying the recursive least squares algorithm directly to the pseudo-regression

$$e_n(\underline{\theta}) = y_n - s_n(\underline{\theta}) = y_n - \underline{\varphi}_n'(\underline{\theta})\underline{\theta}$$

where $\underline{\varphi}_n(\underline{\theta}) = (-s_{n-1}(\underline{\theta}) \ldots -s_{n-n_a}(\underline{\theta}) \; u_{n-1} \ldots u_{n-n_b})'$. Elsewhere Solo (1978,1980), the author, has called this a regression recursion.

In Appendix A it is pointed out that provided

$$u_n \text{ is persistently exciting of order } n_a + n_b \tag{4a}$$

then

$$\lim n^{-1}\sum_1^n \underline{\varphi}_k(\underline{\theta}_0)\underline{\varphi}_k'(\underline{\theta}_0) = \underline{R} \tag{4b}$$

is positive definite.

Now we modify the symmetric IV recursion in much the same way that AML is a modified form of RML, with a priori quantities replaced by a posteriori ones (see Solo, 1979). Thus introduce the recursion

$$\underline{\theta}_n = \underline{\theta}_{n-1} + \underline{P}_n \hat{\underline{\varphi}}_n e_n \tag{5a}$$

$$\underline{P}_n^{-1} = \underline{P}_{n-1}^{-1} + \hat{\underline{\varphi}}_n \hat{\underline{\varphi}}_n' \tag{5b}$$

$$e_n = y_n - \hat{\underline{\varphi}}_n' \underline{\theta}_{n-1} \tag{5c}$$

where $$\hat{\underline{\varphi}}_n' = (-\hat{s}_{n-1}\cdots-\hat{s}_{n-n_a}\ u_{n-1}\cdots u_{n-n_b}) \tag{5d}$$

and $$\hat{s}_n = \hat{\underline{\varphi}}_n' \underline{\theta}_n \ . \tag{5e}$$

The a posteriori residual is

$$\eta_n = y_n - \hat{s}_n = y_n - \hat{\underline{\varphi}}_n' \underline{\theta}_n \ . \tag{5f}$$

Also $$\eta_n = y_n - \hat{\underline{\varphi}}_n' \underline{\theta}_{n-1} - \hat{\underline{\varphi}}_n' \underline{P}_n \hat{\underline{\varphi}}_n e_n = e_n(1 - \hat{\underline{\varphi}}_n' \underline{P}_n \hat{\underline{\varphi}}_n) \ . \tag{5g}$$

It is assumed that the recursion is initialised so that $\underline{P}_p^{-1}$ is positive definite ($p = n_a + n_b$) then by (5b) $\underline{P}_n^{-1}$ is positive definite for all $n \geq p$. This recursion will be called an aposteriori regression recursion (denoted by RR_+): AML of Solo (1979) is also an RR_+. The remainder of the paper is organized as follows: Section II contains the proof. Section III contains some conclusions. There is one Appendix. In the sequel the Appendices of Solo (1979) (henceforth denoted S) will be referred to. Appendix I contains a Martingale convergence theorem (MGCT*) as well as some consequences of the *Matrix Intersion Lemma (MIL)*. Appendix II discusses the positive real lemma.

II. Proof of Convergence

(a) Preliminaries. The structure of the proof is briefly outlined. It is similar to that in S: there are three stages. Firstly consider the following stability property of this RR_+ recursion. In (5a) form $\underline{\theta}_n' \underline{P}_n^{-1} \underline{\theta}_n$ and sum up (cf S, eq (13a)) then use (5g) to find

$$\underline{\theta}_n' \underline{P}_n^{-1} \underline{\theta}_n + \textstyle\sum_1^n \eta_\kappa^2 /(1 - \hat{\underline{\varphi}}_k' \underline{P}_k \hat{\underline{\varphi}}_k) = \textstyle\sum_1^n y_k^2 \quad . \tag{6}$$

Now (1), (2), (3) and the assumption that $\underline{\theta}_0 \in R$ ensure

$$\overline{\lim}\ n^{-1} \textstyle\sum_1^n s_k^2 < \infty \tag{7a}$$

$$\overline{\lim}\ n^{-1} \textstyle\sum_1^n y_k^2 < \infty \qquad \text{a.s.} \tag{7b}$$

Thus, since via MIL (A3) of S, $1 - \hat{\underline{\phi}}_k' \underline{P}_k \hat{\underline{\varphi}}_k < 1$ (this follows since by construction $\underline{P}_k$ is positive definite while $y_k = \infty$, $u_k = \infty$ are zero probability events) we conclude

$$\overline{\lim}\ n^{-1} \textstyle\sum_1^n \eta_k^2 < \infty \qquad \text{a.s.} \tag{8a}$$

and also via (5g)

$$\overline{\lim}\ n^{-1} \textstyle\sum_1^n \hat{s}_k^2 < \infty \qquad \text{a.s.} \tag{8b}$$

Thus purely because a posteriori quantities are used the recursion never becomes unstable in that the power in the forecast errors is bounded.

The second stage in the proof is to suppose a strict positive real condition for $h(L) = (1+A_0(L))^{-1} - \frac{1}{2}$ namely

$$\text{for some small } \delta > 0 \quad , \quad \operatorname{Re} h(e^{i\omega}) \geq \delta \quad . \tag{9}$$

This will yield via (8) and the Martingale Convergence Theorem (MGCT*)

$$\lim n^{-1} \textstyle\sum_1^n (\eta_k - \epsilon_k)^2 = 0 \qquad \text{a.s.} \tag{10a}$$

$$(\underline{\theta}_n - \underline{\theta}_0)' \underline{P}_n^{-1} (\underline{\theta}_n - \underline{\theta}_0)/n \to 0 \qquad \text{a.s.} \tag{10b}$$

Thus the addition of (9) to the assumptions (1), (2), (3) and $\underline{\theta}_0 \in R$ yields an improvement of (8) to

$$\lim n^{-1} \sum_1^n \eta_k^2 = \sigma^2 \quad . \tag{10c}$$

This follows from (10a), (2) and the Cauchy-Schwarz inequality. Next observe that since $\eta_k = y_k - \hat{s}_k = \epsilon_k + s_k - \hat{s}_k$ (10a) can be rewritten

$$n^{-1} \sum_1^n (s_k - \hat{s}_k)^2 \to 0 \qquad \text{a.s.} \tag{10d}$$

Now, however, it follows that

$$n^{-1} \underline{P}_n^{-1} \to \underline{R} \qquad \text{a.s.} \tag{10e}$$

To see this it is only necessary to show

$$n^{-1} \sum_{r \wedge s}^n (s_{k-r} - \hat{s}_{k-r}) u_{k-s} \to 0 \qquad r = 1..n_a \; ; \; s = 1..n_b$$

$$n^{-1} \sum_r^n (s_k - \hat{s}_k) s_{k-r} \to 0 \qquad r = 1..n_a$$

where $r \wedge s = \min(r,s)$. But these follow via the Cauchy-Schwarz inequality from (10c), (3), (7a) and the fact that (3) and (7a) imply

$$u_n^2/n \to 0 \ , \ s_n^2/n \to 0 \ \text{ respectively} \quad .$$

(This follows from MIL (A5) Of S: put $\underline{x}_n = u_n$ and then $\underline{x}_n = s_n$.)

For the final stage of the proof observe that the persistently exciting condition (4a) ensures $\underline{R}$ is positive definite. Then (10b), (10d) yield

$$\underline{\theta}_n \to \underline{\theta}_0 \qquad \text{a.s.} \tag{11}$$

We now proceed to fill in the details of the proof.

Remark. It will also follow as in S that n^{-1} in (10a) can be replaced by $n^{-\nu}$ for some small $\nu > 0$. Also (11) can be improved to

$$n^{\frac{1}{2}-\nu}(\underline{\theta}_n - \underline{\theta}_0) \to \underline{0} \qquad \text{a.s.}$$

(b) Proof details. Rewrite (5a) as

$$\underline{\theta}_n = \underline{\theta}_{n-1} + \underline{P}_{n-1}\, \hat{\underline{\varphi}}_n\, \eta_n \quad . \tag{12a}$$

This follows from (5g) and from MIL which implies

$$\underline{P}_n\, \hat{\underline{\varphi}}_n\, e_n = \underline{P}_{n-1}\, \hat{\underline{\varphi}}_n\, e_n/(1 + \hat{\underline{\varphi}}_n{}'\, \underline{P}_n\, \hat{\underline{\varphi}}_n)$$

$$= \underline{P}_{n-1}\, \hat{\underline{\varphi}}_n\, e_n(1 - \hat{\underline{\varphi}}_n{}'\, \underline{P}_n\, \hat{\underline{\varphi}}_n) \quad .$$

Next denote $T_n = (\underline{\theta}_n - \underline{\theta}_0)'\underline{P}_{n-1}^{-1}(\underline{\theta}_n - \underline{\theta}_0)$ and rewrite (12a) as

$$\underline{P}_{n-1}^{-1}(\underline{\theta}_{n-1} - \underline{\theta}_0) = \underline{P}_{n-1}^{-1}(\underline{\theta}_n - \underline{\theta}_0) - \hat{\underline{\varphi}}_n\, \eta_n \quad .$$

Then, an easy calculation shows

$$T_{n-1} = T_n - (\hat{\underline{\varphi}}_n{}'(\underline{\theta}_n - \underline{\theta}_0))^2 - 2\eta_n(\underline{\theta}_n - \underline{\theta}_0)'\hat{\underline{\varphi}}_n + \hat{\underline{\varphi}}_n{}'\, \underline{P}_{n-1}\, \hat{\underline{\varphi}}_n\, \eta_n^2 \quad . \tag{12b}$$

Next consider that

$$s_n = \underline{\varphi}_n{}'\, \underline{\theta}_0$$

$$\hat{s}_n = \hat{\underline{\varphi}}_n{}'\, \underline{\theta}_n$$

where $s_n = s_n(\underline{\theta}_0)$ and $\underline{\varphi}_n = \underline{\varphi}_n(\underline{\theta}_0)$. Thus

$$s_n - \hat{s}_n = -A_0(L)(s_n - \hat{s}_n) - \hat{\underline{\varphi}}_n{}'(\underline{\theta}_n - \underline{\theta}_0)$$

or

$$(1 + A_0(L))(s_n - \hat{s}_n) = -\hat{\underline{\varphi}}_n{}'(\underline{\theta}_n - \underline{\theta}_0) \quad . \tag{13}$$

However $\eta_n - \epsilon_n = y_n - \hat{s}_n - \epsilon_n = s_n - \hat{s}_n$

so

$$(1 + A_0(L))(\eta_n - \epsilon_n) = -\hat{\underline{\varphi}}_n{}'(\underline{\theta}_n - \underline{\theta}_0) \quad .$$

Letting u,y be generic symbols for input/output sequences respectively call

$$\hat{u}_n = -\hat{\underline{\varphi}}_n'(\underline{\theta}_n - \underline{\theta}_0)$$

$$\hat{y}_n = \eta_n - \epsilon_n + \frac{1}{2}\hat{\underline{\varphi}}_n'(\underline{\theta}_n - \underline{\theta}_0) \quad .$$

Thus $(1 + A_0(L))(\hat{y}_n + \frac{1}{2}\hat{u}_n) = \hat{u}_n$ or

$$\hat{y}_n = [(1 + A_0(L))^{-1} - \frac{1}{2}]\hat{u}_n \quad .$$

Now express (12b) in terms of $\hat{y}_n$, $\hat{u}_n$; note from MIL (A3) that

$$\hat{\underline{\varphi}}_n' \underline{P}_{n-1} \hat{\underline{\varphi}}_n = \hat{\underline{\varphi}}_n' \underline{P}_n \hat{\underline{\varphi}}_n/(1 - \hat{\underline{\varphi}}_n' \underline{P}_n \hat{\underline{\varphi}}_n)$$

and observe that $e_n - \epsilon_n$ depends only on history up to time $n - 1$ (i.e., is F_{n-1} measurable). Then taking conditional expectations yields (cf S)

$$E(T_n + 2\hat{u}_n \hat{y}_n | F_{n-1}) \leq T_{n-1} + 2\sigma^2 \hat{\underline{\varphi}}_n' \underline{P}_n \hat{\underline{\varphi}}_n \quad . \tag{14a}$$

Next introduce $c_n' = 2 \sum_1^n \hat{u}_k \hat{y}_k$. Then according to the strict positive real assumption (9) there is a constant c depending only on initial conditions such that

$$c_n = c_n' + c \geq 2 \delta \sum_1^n \hat{u}_k^2 \geq 0 \tag{15}$$

(see Appendix II of S). Thus in (14a) denoting $T_n' = T_n + c_n$

$$E(T_n' | F_{n-1}) \leq T'_{n-1} + 2\sigma^2 \hat{\underline{\varphi}}_n' \underline{P}_n \hat{\underline{\varphi}}_n \quad . \tag{14b}$$

Now we could proceed as in S by dividing by n: here, however, a slightly different route is taken. Divide through (14b) by $r_n = tr(\underline{P}_n^{-1})$.

$$E(T_n'/r_n | F_{n-1}) \leq T'_{n-1}/r_{n-1} - (T'_{n-1}/r_{n-1})dr_n/r_n + 2\sigma^2 \hat{\underline{\varphi}}_n' \underline{P}_n \hat{\underline{\varphi}}_n/r_n$$

where $dr_n = r_n - r_{n-1}$: notice that r_n is F_{n-1} measurable.

Now bearing in mind MGCT* consider

$$\sum_p^\infty \hat{\varphi}_n' \underline{P}_n \hat{\varphi}_n/r_n = \sum_p^\infty \hat{\varphi}_n' \underline{P}_n \underline{P}_n^{-1} \underline{P}_n \hat{\varphi}_n/r_n$$

$$\leq \sum_p^\infty \hat{\varphi}_n' \underline{P}_n^2 \hat{\varphi}_n$$

$$< \infty \quad \text{by MIL (A4)} \quad .$$

Thus according to MGCT*

$$T_n'/r_n = (T_n + c_n)/r_n \quad \text{converges} \qquad \text{a.s.}$$

$$\sum_p^\infty (T_{n-1}'/r_n) dr_n/r_n < \infty \qquad \text{a.s.}$$

Next observe that

$$r_n = \operatorname{tr}(\underline{P}_n^{-1}) = \sum_p^n \hat{\varphi}_k' \hat{\varphi}_k > \sum_p^n u_k^2 \uparrow \infty \quad .$$

Thus $$\sum_p^\infty dr_n/r_n = \infty \quad ,$$

so we conclude

$$T_n'/r_n = (T_n + c_n)/r_n \to 0 \qquad \text{a.s.} \tag{16}$$

Now observe by (3), (8b) that

$$\overline{\lim}\ r_n/n < \infty \quad .$$

Thus

$$\overline{\lim}\ T_n'/n \leq \overline{\lim}\ T_n'/r_n \ \overline{\lim}\ r_n/n = 0 \quad .$$

Now positivity of T_n, c_n implies (10b)

$$T_n/n \to 0$$

$$c_n/n \to 0$$

or $$\sum_1^n \hat{u}_k \hat{y}_k/n \to 0$$

whereupon condition (9), via (15) yields

$$\sum_1^n \hat{u}_k^2/n \to 0 \quad . \tag{17a}$$

Also the stability of $h(L)$ implies, (as in S, eq. (18))

$$\sum_1^n \hat{y}_k^2 \le \sup_\omega |h(e^{i\omega}|^2 \sum_1^n \hat{u}_k^2$$

so (17a) implies

$$\sum_1^n \hat{y}_k^2/n \to 0 \qquad \text{a.s.} \tag{17b}$$

Finally since $n_k - \epsilon_k = \hat{y}_k + \frac{1}{2}\hat{u}_k$ we obtain

$$n^{-1} \sum_1^n(\eta_k - \epsilon_k)^2 \to 0 \quad . \tag{10a}$$

The proof is complete.

Conclusions. It has been shown that provided conditions (1), (2), (3), that $\underline{\theta}_0 \in R \subset S$ of (1), (4a) and the positive real condition (9) holds then the a posteriori Regression Recursion, for the parameters of a TFWN model, converges without the need of monitoring. This is a rather disappointing result in that the positive real condition must be satisfied by a quantity that depends on the system characteristic function. Now it is clear that the original IV recursion could not be expected to converge unless the positive real condition holds; further even if this holds the IV recursion would in general not converge without being monitored. At least in that respect the present recursion is superior. Finally it may be remarked that in the off line case it it well known that the IV estimator (or its symmetric form) converges even if ϵ_n is a coloured noise. It would be interesting to know if that holds too with the recursive estimator.

Appendix. Persistent Exatation and Positive Definiteness.

The aim is to show that if (4a) holds then we cannot find a fixed $(n_a + n_b)$ vector $\underline{a}$ with $\underline{a}'\ \underline{R}\ \underline{a} = 0$. Suppose there is a vector $\underline{a}$ with $\underline{a}'\ \underline{R}\ \underline{a} = 0$. Partition $\underline{a}$ into $(\underline{\alpha}'\underline{\beta}')$ with $\underline{\alpha}$ an n_a-vector, $\underline{\beta}$ an n_b-vector: also form polynomials

$$\alpha(L) = \alpha_1 L + \ldots + \alpha_{n_a} L^{n_a};\ \beta(L) = \beta_1 L + \ldots + \beta_{n_b} L^{n_b} \quad .$$

Now $\underline{a}'\ \underline{R}\ \underline{a} = 0$ implies

$$\lim n^{-1} \sum_1^n (\underline{a}'\ \underline{\varphi}_k(\underline{\theta}_0))^2 = 0 \quad . \tag{A1}$$

Consider that

$$\underline{a}'\ \underline{\varphi}_k(\underline{\theta}_0) = -\alpha(L)s_k + \beta(L)u_k$$

$$= -\frac{\alpha(L)\theta_0(L)}{1+A_0(L)}\ u_k + \beta(L)u_k$$

$$= \frac{\tau(L)}{1+A_0(L)}\ u_k$$

where $\tau(L) = -\alpha(L)\beta_0(L) + (1 + A_0(L))\beta(L)$. *Notice that* $\tau(L)$ *is of the form* $\tau_1 L + \ldots + \tau_{n_a+n_b} L^{n_a+n_b}$. Consider that

$$\sum_1^n (\tau(L)u_k)^2 = \sum_1^n \left[1 + A_0(L)\left(\frac{\tau(L)}{1+A_0(L)}\ u_k\right)\right]^2$$

(cf S, eq.(18))
$$\leq \sup_\omega |1 + A_0(e^{i\omega})|^2 \sum_1^n \left(\frac{\tau(L)}{1+A_0(L)}\ u_k\right)^2$$

$$= \sup_\omega |1 + A_0(e^{i\omega})|^2 \sum_1^n (\underline{a}'\ \underline{\varphi}_k(\underline{\theta}_0))^2 \quad .$$

Thus since the coefficients of $A_0(L)$ are bounded (A1) implies

$$\lim n^{-1} \sum_1^n (\tau(L)u_k)^2 = 0$$

which contradicts (4a).

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